Modeling the Effects of Fentanyl and Narcan on the Opioid Epidemic in Allegheny County Using Mathematics

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Abstract
In collaboration with the Allegheny County Department of Human Services (DHS), we developed a comprehensive mathematical model to describe the opioid epidemic in Allegheny County. The model is a system of differential equations describing how the size of each population class—Susceptible, Prescribed, Addicted, and Recovered—is changing over time. Variables describing the presence of fentanyl (a synthetic opioid) and the use of Narcan (medication used to block the effects of opioids) were included in the model. Model parameters were estimated using data provided by the DHS and reflect the opioid addiction and overdose rates in Allegheny County. Model simulations highlight the impact of fentanyl and Narcan on the annual overdose deaths. Additional results show the extent to which an increase in the availability of Narcan can decrease opioid-related fatalities over time.

Introduction

Timeline of the Opioid Epidemic

Physicians start prescribing opioids at an increased rate to treat pain.

Annual opioid prescription rate peaks at 81.3 prescriptions per 100 persons.

Overdose deaths increase substantially due to the use of synthetic opioids, such as fentanyl.

The opioid epidemic is officially declared a National Emergency.

Opioid Epidemic in Allegheny County

Timeline of the Opioid Epidemic

1990
2012
2015
2017

The Impact of Fentanyl

Since fentanyl is 80-100 times more powerful than morphine, addicts and dealers often mix it with other opioids to increase their potency and users are more likely to suffer from an overdose.

The Role of Narcan

• Narcan is medication used to block the effects of opioids.
• In 2017, PA increased funding to combat the opioid epidemic, allocating $5 million for Narcan distribution.
• In 2018, the number of prescriptions for Narcan doubled and overdose fatalities decreased.

Community Partner

We collaborated with Peter Jhon, a data analyst from the Allegheny County DHS, to develop a comprehensive mathematical model of the opioid epidemic in Allegheny County.

Mathematical Model

Variables
The population is divided into four classes:
S(t) = proportion of the population that is not using opioids or actively recovering from addiction
P(t) = proportion of the population that is using opioids as prescribed
A(t) = proportion of the population that is addicted to opioids
R(t) = proportion of the population that is in treatment for opioid addiction

* At all times t, S + P + A + R = 1.

Model
Movement between the classes is illustrated with arrows in the diagram below. Green arrows represent death, which occurs at a natural rate (γ) or at an increased rate due to addiction (μP, μA).
The presence of fentanyl (F) increases the death rate of addicts, whereas the availability of Narcan (n) decreases the death rate of addicts.

Figure 3: Mathematical model describing the opioid epidemic in Allegheny County.

Parameters
Table 1: Values of parameters used in model simulations. Orange rows denote local estimates obtained from community partner.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>Death rate</td>
<td>0.2</td>
<td>Vowles et al. (2015), Shah et al. (2017)</td>
</tr>
<tr>
<td>μP</td>
<td>Opioid addiction rate</td>
<td>1.0</td>
<td>Battista et al. (2019)</td>
</tr>
<tr>
<td>μA</td>
<td>Narcan-induced addiction rate</td>
<td>0.007708</td>
<td>Allegheny County DHS</td>
</tr>
<tr>
<td>a</td>
<td>Illicit addiction rate</td>
<td>0.00000136</td>
<td>Battista et al. (2019)</td>
</tr>
<tr>
<td>n</td>
<td>Narcan availability</td>
<td>0.021812</td>
<td>Allegheny County DHS</td>
</tr>
<tr>
<td>R0</td>
<td>Initial proportion of population</td>
<td>0.04</td>
<td>Allegheny County DHS</td>
</tr>
</tbody>
</table>

Equations
Based on the diagram above, the following system of differential equations describes the opioid epidemic in Allegheny County.

\[ \frac{dS}{dt} = -\gamma S - \mu_P SP + \delta R + \delta + b \]
\[ \frac{dP}{dt} = a S - \gamma P - \mu_P SP - \mu_A AP - \mu_P \Delta + \delta + \beta P + \alpha A \]
\[ \frac{dA}{dt} = -\gamma A - (\mu_P + \mu_A) A + (1 - n) \Delta + \beta P + \alpha A - \mu_P \Delta + \delta + \beta P + \alpha A \]
\[ \frac{dR}{dt} = \gamma (S + P + A) - \gamma R - \mu_A A - \mu_P \Delta + \delta + \beta P + \alpha A \]

Simulations and Results

Equations (1)–(4) were solved numerically and results were graphed using the statistical software R. The package deSolve was used to solve the equations with the following initial conditions:
\[ S(0) = 0.990504, \quad P(0) = 0.0008, \quad A(0) = 0.001136, \quad R(0) = 0.000136 \]

Results

Optimal Values of Fentanyl and Narcan Parameters

We used the model to output the predicted annual overdose death rate in Allegheny County from 2004 to 2018 for different values of F and n. Model predictions best matched the observed death rate when 47% of opioid-related additions involved fentanyl (F = 0.47) and Narcan availability increased from n = 0 to n = 0.57 starting in 2017.

Figure 4: Model solutions show the values of S (blue), P (red), A (green), and R (purple) over time.

Long-term Impact of Increasing Use of Fentanyl

We simulated the model to predict the cumulative number of opioid overdose deaths in Allegheny County over the next decade (2018–2028) for different values of F and n. Results show that as the percentage of drugs containing fentanyl (F) increases, the impact of increasing the availability of Narcan within the community diminishes.

Figure 5: Bar plots show model predictions of the annual number of opioid overdose deaths when (A) the effects of fentanyl and Narcan are ignored (F = 0, n = 0); (B) percent of drugs involving fentanyl is increased to 47% in 2015 (F = 0.47) and the impact of Narcan is ignored (n = 0); (C) percent of drugs involving fentanyl is increased to 47% in 2015 (F = 0.47) and the impact of Narcan is increased from n = 0 to n = 0.57 in 2017.

Conclusions

• Results from model simulations indicate that increasing the availability of Narcan in the community will result in a meaningful reduction in the cumulative number of overdose deaths over a 10-year period.
• However, an increased presence of fentanyl will render Narcan less effective in reducing overdose deaths.
• State and local policymakers, including Governor Wolf, and organizations, such as the Allegheny DHS, can use our model to inform their decisions about the most beneficial way to allocate future funding to combat the opioid epidemic.

Resources

Allegheny County, (n.d.). Opioid Information. Retrieved from https://www.alleghenycountypa.gov/Heath-Department/Programs/Pages/Opioids-Information.aspx

Acknowledgements

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Figure 6: Graphs display the cumulative number of opioid overdose deaths for three different values of (percentage of drugs containing fentanyl) and Narcan availability over time.